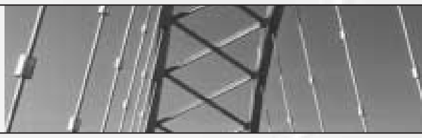


# 8

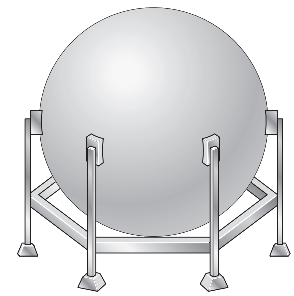
# Applications of Plane Stress (Pressure Vessels, Beams, and Combined Loadings)

## Spherical Pressure Vessels

*When solving the problems for Section 8.2, assume that the given radius or diameter is an inside dimension and that all internal pressures are gage pressures.*

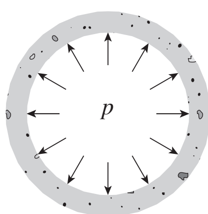
**Problem 8.2-1** A large spherical tank (see figure) contains gas at a pressure of 400 psi. The tank is 45 ft in diameter and is constructed of high-strength steel having a yield stress in tension of 80 ksi.

Determine the required thickness (to the nearest 1/4 inch) of the wall of the tank if a factor of safety of 3.5 with respect to yielding is required.



**Solution 8.2-1** **Spherical tank**

CROSS SECTION




Radius: $r = \frac{1}{2}(45 \text{ ft}) = 270 \text{ in.}$

Internal pressure: $p = 400$ psi

Yield stress: $\sigma_y = 80$ ksi (steel)

Factor of safety: $n = 3.5$

MINIMUM WALL THICKNESS $t_{min}$

From Eq. (8-1): $\sigma_{max} = \frac{pr}{2t}$ or $\frac{\sigma_y}{n} = \frac{pr}{2t}$

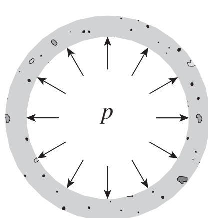
$t = \frac{prn}{2\sigma_y} = 2.36 \text{ in.}$

Use the next higher 1/4 inch: $t_{min} = 2.50$ in. ←

**Problem 8.2-2** Solve the preceding problem if the internal pressure is 3.5 MPa, the diameter is 18 m, the yield stress is 550 MPa, and the factor of safety is 3.0. Determine the required thickness to the nearest millimeter.

**Solution 8.2-2** **Spherical tank**

CROSS SECTION




Radius: $r = \frac{1}{2}(18 \text{ m}) = 9 \text{ m}$

Internal pressure: $p = 3.5$ MPa

Yield stress: $\sigma_y = 550$ MPa

Factor of safety: $n = 3.0$

MINIMUM WALL THICKNESS $t_{min}$

From Eq. (8-1): $\sigma_{max} = \frac{pr}{2t}$ or $\frac{\sigma_x}{n} = \frac{pr}{2t}$

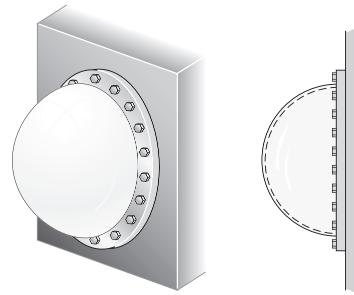
$t = \frac{prn}{2\sigma_y} = 85.9 \text{ mm}$

Use the next higher millimeter:

$t_{min} = 86$ mm ←

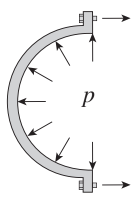
**Problem 8.2-3** A hemispherical window (or *viewport*) in a decompression chamber (see figure) is subjected to an internal air pressure of 80 psi. The port is attached to the wall of the chamber by 18 bolts.

Find the tensile force $F$ in each bolt and the tensile stress $\sigma$ in the viewport if the radius of the hemisphere is 7.0 in. and its thickness is 1.0 in.



## Solution 8.2-3 Hemispherical viewport

FREE-BODY DIAGRAM




Radius: $r = 7.0$ in.
Internal pressure: $p = 80$ psi
Wall thickness: $t = 1.0$ in.
18 bolts

$T$ = total tensile force in 18 bolts

$$\sum_{\text{HORIZ}} = T - pA = 0 \qquad T = pA = p(\pi r^2)$$

$F$ = force in one bolt

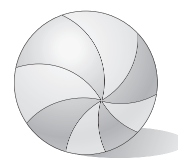
$$F = \frac{T}{18} = \frac{1}{18}(\pi p r^2) = 684 \text{ lb} \longleftarrow$$

TENSILE STRESS IN VIEWPORT (EQ. 8-1)

$$\sigma = \frac{pr}{2t} = 280 \text{ psi} \longleftarrow$$

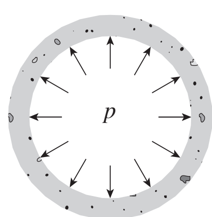
**Problem 8.2-4** A rubber ball (see figure) is inflated to a pressure of 60 kPa. At that pressure the diameter of the ball is 230 mm and the wall thickness is 1.2 mm. The rubber has modulus of elasticity $E = 3.5$ MPa and Poisson's ratio $\nu = 0.45$.

Determine the maximum stress and strain in the ball.



## Solution 8.2-4 Rubber ball

CROSS-SECTION




Radius: $r = (230 \text{ mm})/2 = 115$ mm
Internal pressure: $p = 60$ kPa
Wall thickness: $t = 1.2$ mm
Modulus of elasticity: $E = 3.5$ MPa (rubber)
Poisson's ratio: $\nu = 0.45$ (rubber)

MAXIMUM STRESS (EQ. 8-1)

$$\sigma_{max} = \frac{pr}{2t} = \frac{(60 \text{ kPa})(115 \text{ mm})}{2(1.2 \text{ mm})} = 2.88 \text{ MPa} \longleftarrow$$

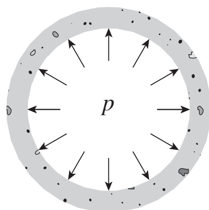
MAXIMUM STRAIN (EQ. 8-4)

$$\varepsilon_{max} = \frac{pr}{2tE}(1 - \nu) = \frac{(60 \text{ kPa})(115 \text{ mm})}{2(1.2 \text{ mm})(3.5 \text{ MPa})}(0.55) = 0.452 \longleftarrow$$

**Problem 8.2-5** Solve the preceding problem if the pressure is 9.0 psi, the diameter is 9.0 in., the wall thickness is 0.05 in., the modulus of elasticity is 500 psi, and Poisson's ratio is 0.45.

**Solution 8.2-5 Rubber ball**

CROSS-SECTION




Radius: $r = \frac{1}{2}(9.0 \text{ in.}) = 4.5 \text{ in.}$

Internal pressure: $p = 9.0$ psi
Wall thickness: $t = 0.05$ in.
Modulus of elasticity: $E = 500$ psi (rubber)
Poisson's ratio: $\nu = 0.45$ (rubber)

MAXIMUM STRESS (EQ. 8-1)

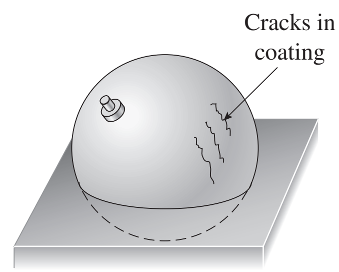
$$\sigma_{max} = \frac{pr}{2t} = \frac{(9.0 \text{ psi})(4.5 \text{ in.})}{2(0.05 \text{ in.})}$$
$$= 405 \text{ psi} \longleftarrow$$

MAXIMUM STRAIN (EQ. 8-4)

$$\varepsilon_{max} = \frac{pr}{2tE}(1 - \nu) = \frac{(9.0 \text{ psi})(4.5 \text{ in.})}{2(0.05 \text{ in.})(500 \text{ psi})}(0.55)$$
$$= 0.446 \longleftarrow$$

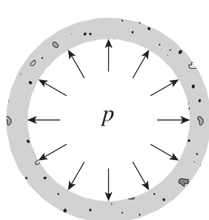
**Problem 8.2-6** A spherical steel pressure vessel (diameter 480 mm, thickness 8.0 mm) is coated with brittle lacquer that cracks when the strain reaches $150 \times 10^{-6}$ (see figure).

What internal pressure $p$ will cause the lacquer to develop cracks? (Assume $E = 205$ GPa and $\nu = 0.30$.)




**Solution 8.2-6 Spherical vessel with brittle coating**

CROSS-SECTION




$r = 240$ mm $E = 205$ GPa (steel) $t = 8.0$ mm
$\nu = 0.30$

Cracks occur when $\varepsilon_{max} = 150 \times 10^{-6}$

From Eq. (8-4): $\varepsilon_{max} = \frac{pr}{2tE}(1 - \nu)$

$$\therefore p = \frac{2tE\,\varepsilon_{max}}{r(1 - \nu)}$$

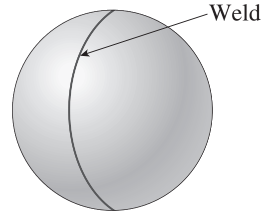
$$p = \frac{2(8.0 \text{ mm})(205 \text{ GPa})(150 \times 10^{-6})}{(240 \text{ mm})(0.70)}$$
$$= 2.93 \text{ MPa} \longleftarrow$$

**Problem 8.2-7** A spherical tank of diameter 50 in. and wall thickness 2.0 in. contains compressed air at a pressure of 2400 psi. The tank is constructed of two hemispheres joined by a welded seam (see figure on the next page).

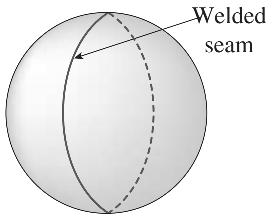
(a) What is the tensile load $f$ (lb per in. of length of weld) carried by the weld? (See the figure on the next page.)

(b) What is the maximum shear stress $\tau_{max}$ in the wall of the tank?

(c) What is the maximum normal strain $\epsilon$ in the wall? (For steel, assume $E = 31 \times 10^6$ psi and $\nu = 0.29$.)




**Solution 8.2-7 Welded tank (spherical)**




$r = 25$ in. $\quad E = 31 \times 10^6$ psi
$t = 2.0$ in. $\quad \nu = 0.29$ (Steel)
$p = 2400$ psi

(a) Tensile load carried by weld

$T$ = total load $\quad f$ = load per inch
$T = pA = p(\pi r^2)$ $\quad c$ = Circumference of tank $= 2\pi r$

$$f = \frac{T}{c} = \frac{p(\pi r^2)}{2\pi r} = \frac{pr}{2} = \frac{(2400 \text{ psi})(25 \text{ in.})}{2}$$
$$= 30.0 \text{ k/in.} \longleftarrow$$

(b) Maximum shear stress in wall (Eq. 8-3)

$$\tau_{max} = \frac{pr}{4t} = \frac{(2400 \text{ psi})(25 \text{ in.})}{4(2 \text{ in.})}$$
$$= 7500 \text{ psi} \longleftarrow$$

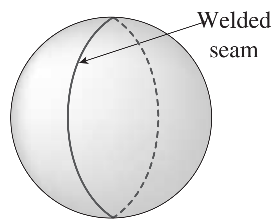
(c) Maximum normal strain in wall (Eq. 8-4)

$$\varepsilon_{max} = \frac{pr(1-\nu)}{2tE} = \frac{(2400 \text{ psi})(25 \text{ in.})(0.71)}{2(2.0 \text{ in.})(31 \times 10^6 \text{ psi})}$$
$$= 344 \times 10^{-6} \longleftarrow$$

**Problem 8.2-8** Solve the preceding problem for the following data: diameter 1.0 m, thickness 50 mm, pressure 24 MPa, modulus 210 GPa, and Poisson's ratio 0.29.

**Solution 8.2-8 Welded tank (spherical)**

$r = 500$ mm $\quad E = 210$ GPa
$t = 50$ mm $\quad \nu = 0.29$ (Steel)
$p = 24$ MPa




(a) Tensile load carried by weld

$T$ = total load $\quad f$ = load per inch
$T = pA = p(\pi r^2)$ $\quad c$ = Circumference of tank $= 2\pi r$

$$f = \frac{T}{c} = \frac{p(\pi r^2)}{2\pi r} = \frac{pr}{2} = \frac{(24 \text{ MPa})(500 \text{ mm})}{2}$$
$$= 6.0 \text{ MN/m} \longleftarrow$$

(b) Maximum shear stress in wall (Eq. 8-3)

$$\tau_{max} = \frac{pr}{4t} = \frac{(24 \text{ MPa})(500 \text{ mm})}{4(50 \text{ mm})}$$
$$= 60.0 \text{ MPa} \longleftarrow$$

(c) Maximum normal strain in wall (Eq. 8-4)

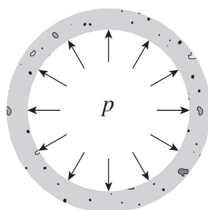
$$\varepsilon_{max} = \frac{pr}{2tE}(1-\nu)$$
$$= \frac{(24 \text{ MPa})(500 \text{ mm})}{2(50 \text{ mm})(210 \times 10^3 \text{ MPa})}(1 - 0.29)$$
$$= 406 \times 10^{-6} \longleftarrow$$

**Problem 8.2-9** A spherical stainless-steel tank having a diameter of 20 in. is used to store propane gas at a pressure of 2400 psi. The properties of the steel are as follows: yield stress in tension, 140,000 psi; yield stress in shear, 65,000 psi; modulus of elasticity, $30 \times 10^6$ psi; and Poisson's ratio, 0.28. The desired factor of safety with respect to yielding is 2.75. Also, the normal strain must not exceed $1000 \times 10^{-6}$.

Determine the minimum permissible thickness $t_{min}$ of the tank.

**Solution 8.2-9** **Propane tank (steel)**

CROSS-SECTION




$r = 10$ in. $\quad E = 30 \times 10^6$ psi

$p = 2400$ psi $\quad \nu = 0.28$

$\sigma_y = 140{,}000$ psi $\quad n = 2.75$

$\tau_y = 65{,}000$ psi $\quad \varepsilon_{max} = 1000 \times 10^{-6}$

MINIMUM WALL THICKNESS $t$

(1) TENSION (EQ. 8-1) $\sigma_{max} = \dfrac{pr}{2t_1}$

$$t_1 = \frac{pr}{2\sigma_{max}} = \frac{pr}{2(\sigma_y/n)} = \frac{(2400 \text{ psi})(10 \text{ in.})}{2(140{,}000 \text{ psi})/2.75}$$

$= 0.236$ in.

(2) SHEAR (EQ. 8-3) $\tau_{max} = \dfrac{pr}{4t_2}$

$$t_2 = \frac{pr}{4\tau_y/n} = \frac{(2400 \text{ psi})(10 \text{ in.})}{4(65{,}000 \text{ psi})/2.75}$$

$= 0.254$ in.

(3) STRAIN (EQ. 8-4) $\varepsilon_{max} = \dfrac{pr}{2t_3E}(1 - \nu)$

$$t_3 = \frac{pr}{2\varepsilon_{max}E}(1 - \nu)$$

$$= \frac{(2400 \text{ psi})(10 \text{ in.})}{2(1000 \times 10^{-6})(30 \times 10^6 \text{ psi})}(1 - 0.28)$$

$= 0.288$ in.

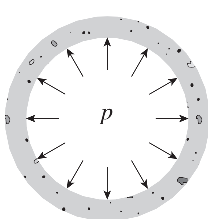
Strain governs since $t_3 > t_1$ and $t_3 > t_2$

$\therefore t_{min} = 0.288$ in. ⟵

**Problem 8.2-10** Solve the preceding problem if the diameter is 500 mm, the pressure is 16 MPa, the yield stress in tension is 950 MPa, the yield stress in shear is 450 MPa, the factor of safety is 2.4, the modulus of elasticity is 200 GPa, Poisson's ratio is 0.28, and the normal strain must not exceed $1200 \times 10^{-6}$.

**Solution 8.2-10** **Propane tank (steel)**

CROSS-SECTION




$r = 250$ mm

$E = 200$ GPa

$p = 16$ MPa

$\nu = 0.28$

$\sigma_y = 950$ MPa

$n = 2.4$

$\tau_y = 450$ MPa

$\varepsilon_{max} = 1200 \times 10^{-6}$

MINIMUM WALL THICKNESS $t$

(1) TENSION (EQ. 8-1) $\sigma_{max} = \dfrac{pr}{2t_1}$

$$t_1 = \frac{pr}{2\sigma_{max}} = \frac{pr}{2(\sigma_y/n)} = \frac{(16 \text{ MPa})(250 \text{ mm})}{2(950 \text{ MPa})/2.4}$$

$= 5.053$ mm

(2) SHEAR (EQ. 8-3) $\tau_{max} = \dfrac{pr}{4t_2}$

$$t_2 = \frac{pr}{4\tau_y/n} = \frac{(16 \text{ MPa})(250 \text{ mm})}{4(450 \text{ MPa})/2.4}$$

$= 5.333$ mm

(3) STRAIN (EQ. 8-4) $\varepsilon_{max} = \dfrac{pr}{2t_3E}(1 - \nu)$

$$t_3 = \frac{pr}{2\varepsilon_{max}E}(1 - \nu)$$

$$= \frac{(16 \text{ MPa})(250 \text{ mm})}{2(1200 \times 10^{-6})(200 \text{ GPa})}(120.28)$$

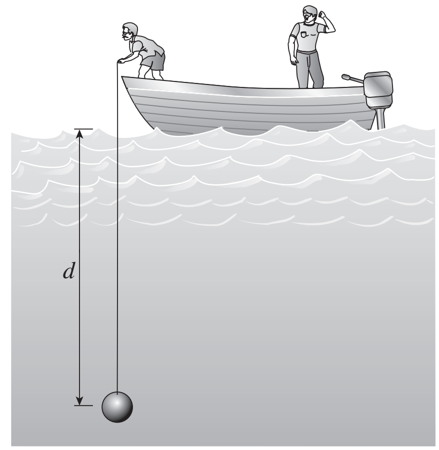
$= 6.000$ mm

Strain governs since $t_3 > t_1$ and $t_3 > t_2$

$\therefore t_{min} = 6.0$ mm ⟵

**Problem 8.2-11** A hollow pressurized sphere having radius $r = 4.8$ in. and wall thickness $t = 0.4$ in. is lowered into a lake (see figure). The compressed air in the tank is at a pressure of 24 psi (gage pressure when the tank is out of the water).

At what depth $D_0$ will the wall of the tank be subjected to a compressive stress of 90 psi?



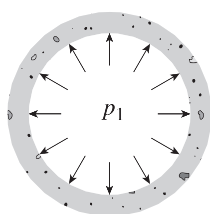

**Solution 8.2-11 Pressurized sphere under water**

CROSS-SECTION

$r = 4.8$ in. $\quad p_1 = 24$ psi

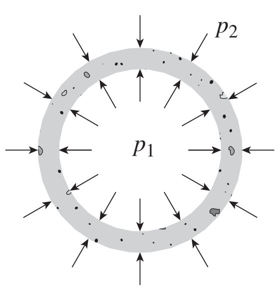
$t = 0.4$ in. $\quad \gamma =$ density of water $= 62.4\ \text{lb/ft}^3$

(1) IN AIR: $p_1 = 24$ psi




(1) IN AIR

(2) UNDER WATER: $p_1 = 24$ psi




(2) UNDER WATER

$D_0 =$ depth of water (in.)

$$p_2 = \gamma D_0 = \left(\frac{62.4\ \text{lb/ft}^3}{1728\ \text{in.}^3/\text{ft}^3}\right) D_0 = 0.036111\ D_0\ \text{(psi)}$$

Compressive stress in tank wall equals 90 psi. (Note: $\sigma$ is positive in tension.)

$$\sigma = \frac{pr}{2t} = \frac{(p_1 - p_2)r}{2t} \qquad \sigma = -90\ \text{psi}$$

$$-90\ \text{psi} = \frac{(24\ \text{psi} - 0.03611\ D_0)(4.8\ \text{in.})}{2(0.4\ \text{in.})}$$

$$= 144 - 0.21667 D_0$$

Solve for $D_0$: $D_0 = \dfrac{234}{0.21667}$

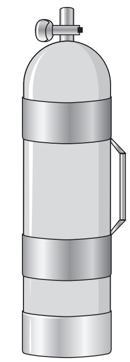
$$= 1080\ \text{in.} = 90\ \text{ft} \longleftarrow$$

## Cylindrical Pressure Vessels

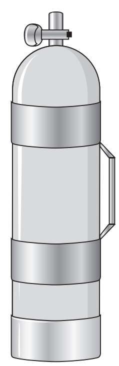
*When solving the problems for Section 8.3, assume that the given radius or diameter is an inside dimension and that all internal pressures are gage pressures.*

**Problem 8.3-1** A scuba tank (see figure) is being designed for an internal pressure of 1600 psi with a factor of safety of 2.0 with respect to yielding. The yield stress of the steel is 35,000 psi in tension and 16,000 psi in shear.

If the diameter of the tank is 7.0 in., what is the minimum required wall thickness?



**Solution 8.3-1** **Scuba tank**



Cylindrical pressure vessel

$p = 1600$ psi $\quad n = 2.0 \quad d = 7.0$ in.

$r = 3.5$ in. $\quad \sigma_y = 35{,}000$ psi $\quad \tau_y = 16{,}000$ psi

$$\sigma_{allow} = \frac{\sigma_y}{n} = 17{,}500 \text{ psi} \qquad \tau_{allow} = \frac{t_y}{n} = 8{,}000 \text{ psi}$$

Find required wall thickness $t$.

(1) BASED ON TENSION (EQ. 8-5) $\quad \sigma_{max} = \frac{pr}{t}$

$$t_1 = \frac{pr}{\sigma_{allow}} = \frac{(1600 \text{ psi})(3.5 \text{ in.})}{17{,}500 \text{ psi}} = 0.320 \text{ in.}$$

(2) BASED ON SHEAR (EQ. 8-10) $\quad \tau_{max} = \frac{pr}{2t}$

$$t_2 = \frac{pr}{2\tau_{allow}} = \frac{(1600 \text{ psi})(3.5 \text{ in.})}{2(8{,}000 \text{ psi})} = 0.350 \text{ in.}$$

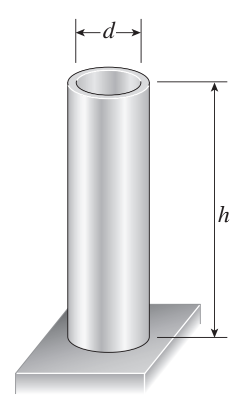
Shear governs since $t_2 > t_1$

$\therefore\ t_{min} = 0.350$ in. ←

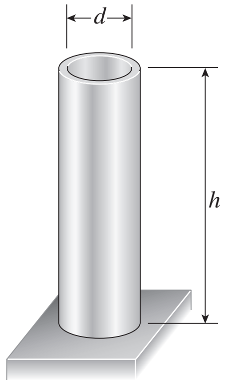
**Problem 8.3-2** A tall standpipe with an open top (see figure) has diameter $d = 2.0$ m and wall thickness $t = 18$ mm.

(a) What height $h$ of water will produce a circumferential stress of 10.9 MPa in the wall of the standpipe?

(b) What is the axial stress in the wall of the tank due to the water pressure?



**Solution 8.3-2** **Vertical standpipe**




$d = 2.0$ m $\quad r = 1.0$ m $\quad t = 18$ mm
$\gamma$ = weight density of water = 9.81 kN/m$^3$
$h$ = height of water (meters)
$p$ = water pressure
$= \gamma h = (9810\ \text{N/m}^3)(h)(10^{-6})$ MPa
$= 0.00981h$ (MPa)

(a) HEIGHT OF WATER: $\sigma_1 = \dfrac{pr}{t}$ (EQ. 8-5)

$\sigma_1 = 10.9$ MPa $\quad 10.9 = \dfrac{(0.00981h)(1.0\ \text{m})}{0.018\ \text{m}}$

Solve for $h$ (meters): $h = \dfrac{(10.9)(0.018)}{(0.00981)(1.0)}$

$= 20.0$ m ←

(b) AXIAL STRESS IN THE WALL DUE TO WATER PRESSURE ALONE

Because the top of the tank is open, the internal pressure of the water produces no axial (longitudinal) stresses in the wall of the tank. Axial stress equals *zero*. ←

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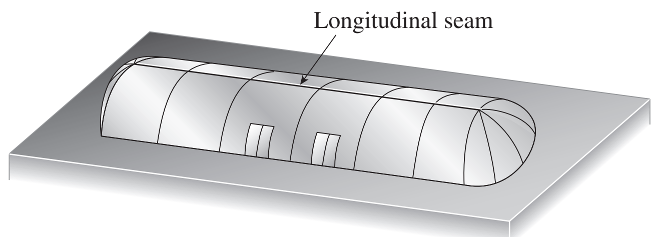
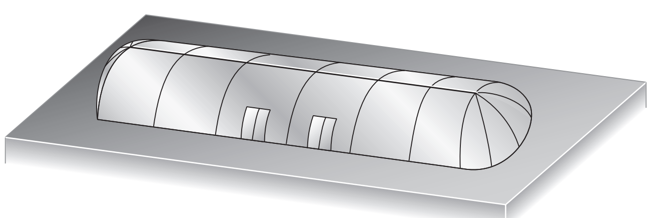
**Problem 8.3-3** An inflatable structure used by a traveling circus has the shape of a half-circular cylinder with closed ends (see figure). The fabric and plastic structure is inflated by a small blower and has a radius of 40 ft when fully inflated. A longitudinal seam runs the entire length of the "ridge" of the structure.

If the longitudinal seam along the ridge tears open when it is subjected to a tensile load of 540 pounds per inch of seam, what is the factor of safety $n$ against tearing when the internal pressure is 0.5 psi and the structure is fully inflated?




**Solution 8.3-3** **Inflatable structure**



Half-circular cylinder
$r = 40$ ft $= 480$ in.
Internal pressure $p = 0.5$ psi

$T$ = tensile force per unit length of longitudinal seam
Seam tears when $T = T_{max} = 540$ lb/in.
Find factor of safety against tearing.

CIRCUMFERENTIAL STRESS (EQ. 8-5)

$\sigma_1 = \dfrac{pr}{t}$ where $t$ = thickness of fabric

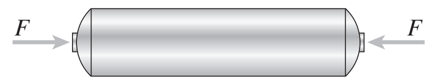
Actual value of $T$ due to internal pressure $= \sigma_1 t$
$\therefore T = \sigma_1 t = pr = (0.5\ \text{psi})(480\ \text{in.}) = 240$ lb/in.

FACTOR OF SAFETY

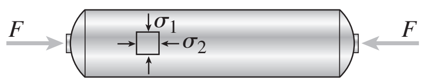
$n = \dfrac{T_{max}}{T} = \dfrac{540\ \text{lb/in.}}{240\ \text{lb/in.}} = 2.25$ ←

**Problem 8.3-4** A thin-walled cylindrical pressure vessel of radius $r$ is subjected simultaneously to internal gas pressure $p$ and a compressive force $F$ acting at the ends (see figure).

What should be the magnitude of the force $F$ in order to produce pure shear in the wall of the cylinder?




**Solution 8.3-4 Cylindrical pressure vessel**




$r$ = Radius

$p$ = Internal pressure

STRESSES (SEE EQS. 8-5 AND 8-6):

$$\sigma_1 = \frac{pr}{t}$$

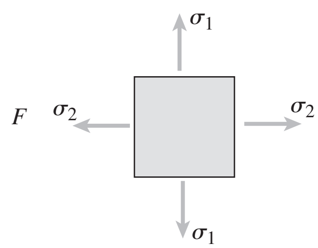
$$\sigma_2 = \frac{pr}{2t} - \frac{F}{A} = \frac{pr}{2t} - \frac{F}{2\pi rt}$$

FOR PURE SHEAR, the stresses $\sigma_1$ and $\sigma_2$ must be equal in magnitude and opposite in sign (see, e.g., Fig. 7-11 in Section 7.3).

$$\therefore \sigma_1 = -\sigma_2$$

OR $$\frac{pr}{t} = -\left(\frac{pr}{2t} - \frac{F}{2\pi rt}\right)$$

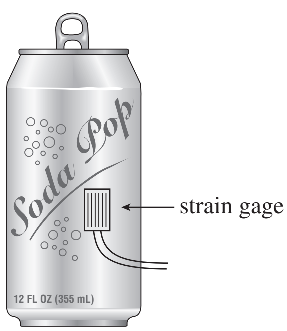
Solve for $F$: $F = 3\pi pr^2$ ←




**Problem 8.3-5** A strain gage is installed in the longitudinal direction on the surface of an aluminum beverage can (see figure). The radius-to-thickness ratio of the can is 200. When the lid of the can is popped open, the strain changes by $\epsilon_0 = 170 \times 10^{-6}$.

What was the internal pressure $p$ in the can? (Assume $E = 10 \times 10^6$ psi and $\nu = 0.33$.)


**Solution 8.3-5 Aluminum can**




$$\frac{r}{t} = 200 \qquad E = 10 \times 10^6 \text{ psi} \qquad \nu = 0.33$$

$\varepsilon_0$ = change in strain when pressure is released $= 170 \times 10^{-6}$

Find internal pressure $p$.

STRAIN IN LONGITUDINAL DIRECTION (EQ. 8-11a)

$$\varepsilon_2 = \frac{pr}{2tE}(1 - 2\nu) \quad \text{or} \quad p = \frac{2tE\varepsilon_2}{r(1 - 2\nu)}$$

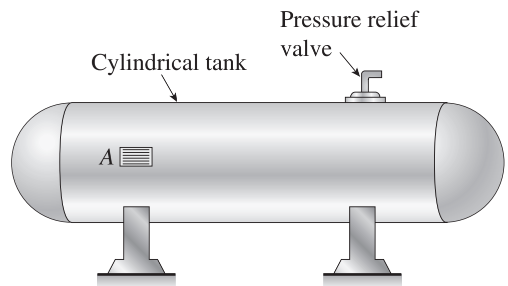
$$\varepsilon_2 = \varepsilon_0 \quad \therefore \quad p = \frac{2tE\varepsilon_0}{(r)(1 - 2\nu)} = \frac{2E\varepsilon_0}{(r/t)(1 - 2\nu)}$$

Substitute numerical values:

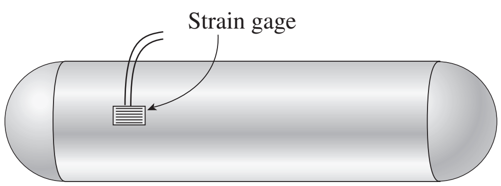
$$p = \frac{2(10 \times 10^6 \text{ psi})(170 \times 10^{-6})}{(200)(1 - 0.66)} = 50 \text{ psi}$$ ←

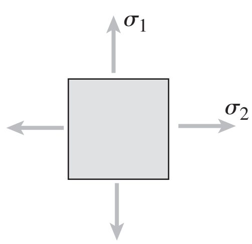
**Problem 8.3-6** A circular cylindrical steel tank (see figure) contains a volatile fuel under pressure. A strain gage at point $A$ records the longitudinal strain in the tank and transmits this information to a control room. The ultimate shear stress in the wall of the tank is 82 MPa and a factor of safety of 2 is required.

At what value of the strain should the operators take action to reduce the pressure in the tank? (Data for the steel are as follows: modulus of elasticity $E = 205$ GPa and Poisson's ratio $\nu = 0.30$.)




**Solution 8.3-6** **Cylindrical tank**




$\tau_{ULT} = 82$ MPa $\quad E = 205$ GPa $\quad \nu = 0.30$

$n = 2$ (factor of safety) $\quad \tau_{max} = \dfrac{\tau_{ULT}}{n} = 41$ MPa

Find maximum allowable strain reading at the gage.

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

From Eq. (8-10):

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{pr}{2t} \qquad \therefore \quad p_{max} = \frac{2t\tau_{max}}{r}$$

From Eq. (8-11a): $\varepsilon_2 = \dfrac{pr}{2tE}(1 - 2\nu)$

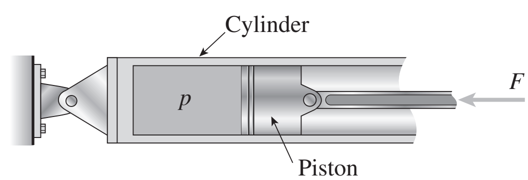
$$(\varepsilon_2)_{max} = \frac{p_{max}r}{2tE}(1 - 2\nu) = \frac{\tau_{max}}{E}(1 - 2\nu)$$

$$\varepsilon_{max} = \frac{41 \text{ MPa}}{205 \text{ GPa}}(1 - 0.60) = 80 \times 10^{-6} \quad \longleftarrow$$

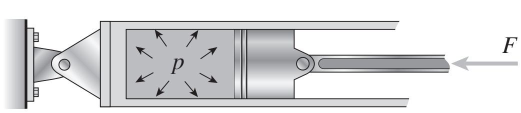
---

**Problem 8.3-7** A cylinder filled with oil is under pressure from a piston, as shown in the figure. The diameter $d$ of the piston is 1.80 in. and the compressive force $F$ is 3500 lb. The maximum allowable shear stress $\tau_{allow}$ in the wall of the cylinder is 5500 psi.

What is the minimum permissible thickness $t_{min}$ of the cylinder wall? (See the figure on the next page.)




**Solution 8.3-7** **Cylinder with internal pressure**




$d = 1.80$ in. $\quad r = 0.90$ in.
$F = 3500$ lb $\quad \tau_{allow} = 5500$ psi

Find minimum thickness $t_{min}$.

Pressure in cylinder: $p = \dfrac{F}{A} = \dfrac{F}{\pi r^2}$

Maximum shear stress (Eq. 8-10):

$$\tau_{max} = \frac{pr}{2t} = \frac{F}{2\pi rt}$$

Minimum thickness:

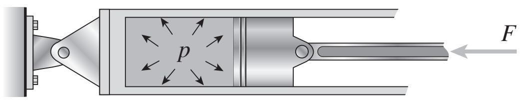
$$t_{min} = \frac{F}{2\pi r \tau_{allow}}$$

Substitute numerical values:

$$t_{min} = \frac{3500 \text{ lb}}{2\pi(0.90 \text{ in.})(5500 \text{ psi})} = 0.113 \text{ in.} \quad \longleftarrow$$

**Problem 8.3-8** Solve the preceding problem if $d = 90$ mm, $F = 42$ kN, and $\tau_{allow} = 40$ MPa.

**Solution 8.3-8 Cylinder with internal pressure**




$d = 90$ mm $\quad r = 45$ mm
$F = 42.0$ kN $\quad \tau_{allow} = 40$ MPa

Find minimum thickness $t_{min}$.

Pressure in cylinder: $p = \dfrac{F}{A} = \dfrac{F}{\pi r^2}$

Maximum shear stress (Eq. 8-10):

$$\tau_{max} = \frac{pr}{2t} = \frac{F}{2\pi rt}$$

Minimum thickness:

$$t_{min} = \frac{F}{2\pi r \tau_{allow}}$$

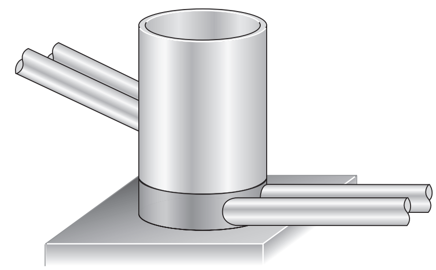
Substitute numerical values:

$$t_{min} = \frac{42.0 \text{ kN}}{2\pi(45 \text{ mm})(40 \text{ MPa})} = 3.71 \text{ mm} \longleftarrow$$

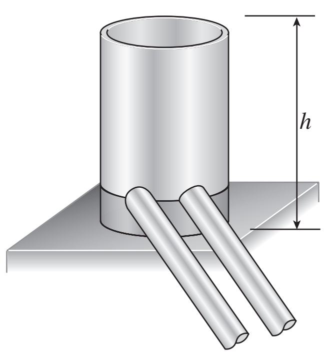
**Problem 8.3-9** A standpipe in a water-supply system (see figure) is 12 ft in diameter and 6 inches thick. Two horizontal pipes carry water out of the standpipe; each is 2 ft in diameter and 1 inch thick. When the system is shut down and water fills the pipes but is not moving, the hoop stress at the bottom of the standpipe is 130 psi.

(a) What is the height $h$ of the water in the standpipe?

(b) If the bottoms of the pipes are at the same elevation as the bottom of the standpipe, what is the hoop stress in the pipes?



**Solution 8.3-9 Vertical standpipe**




$d = 12$ ft $= 144$ in. $\quad r = 72$ in. $\quad t = 6$ in.

$\gamma = 62.4 \text{ lb/ft}^3 = \dfrac{62.4}{1728} \text{ lb/in.}^3$

$\sigma_1$ = hoop stress at bottom of standpipe = 130 psi

(a) FIND HEIGHT $h$ OF WATER IN THE STANDPIPE

$p$ = pressure at bottom of standpipe = $\gamma h$

From Eq. (8-5): $\sigma_1 = \dfrac{pr}{t} = \dfrac{\gamma hr}{t} \quad$ or $\quad h = \dfrac{\sigma_1 t}{\gamma r}$

Substitute numerical values:

$$h = \frac{(130 \text{ psi})(6 \text{ in.})}{\left(\dfrac{62.4 \text{ lb}}{1728 \text{ in.}^3}\right)(72 \text{ in.})} = 300 \text{ in.} = 25 \text{ ft} \longleftarrow$$

HORIZONTAL PIPES

$d_1 = 2$ ft $= 24$ in. $\quad r_1 = 12$ in. $\quad t_1 = 1.0$ in.

(b) FIND HOOP STRESS $\sigma_1$ IN THE PIPES

Since the pipes are 2 ft in diameter, the depth of water to the center of the pipes is about 24 ft.

$h_1 = 24$ ft $= 288$ in. $\quad p_1 = \gamma h_1$

$$\sigma_1 = \frac{p_1 r_1}{t_1} = \frac{\gamma h_1 r_1}{t_1} = \frac{\left(\dfrac{62.4 \text{ lb}}{1728 \text{ in.}^3}\right)(288 \text{ in.})(12 \text{ in.})}{1.0 \text{ in.}}$$
$$= 125 \text{ psi}$$

Based on the average pressure in the pipes:

$\sigma_1 \approx 125$ psi $\longleftarrow$

**Problem 8.3-10** A cylindrical tank with hemispherical heads is constructed of steel sections that are welded circumferentially (see figure). The tank diameter is 1.2 m, the wall thickness is 20 mm, and the internal pressure is 1600 kPa.

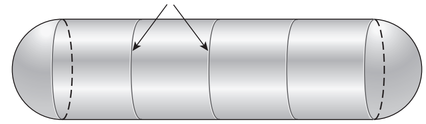
(a) Determine the maximum tensile stress $\sigma_h$ in the heads of the tank.

(b) Determine the maximum tensile stress $\sigma_c$ in the cylindrical part of the tank.

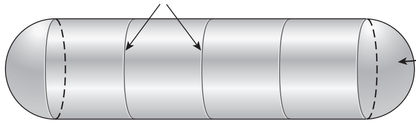
(c) Determine the tensile stress $\sigma_w$ acting perpendicular to the welded joints.

(d) Determine the maximum shear stress $\tau_h$ in the heads of the tank.

(e) Determine the maximum shear stress $\tau_c$ in the cylindrical part of the tank.




**Solution 8.3-10** **Cylindrical tank**




$d = 1.2$ m $\quad r = 0.6$ m

$t = 20$ mm $\quad p = 1600$ kPa

(a) Maximum tensile stress in hemispheres (Eq. 8-1)

$$\sigma_h = \frac{pr}{2t} = \frac{(1600 \text{ kPa})(0.6 \text{ m})}{2(20 \text{ mm})} = 24.0 \text{ MPa} \quad \longleftarrow$$

(b) Maximum stress in cylinder (Eq. 8-5)

$$\sigma_c = \frac{pr}{t} = 2\sigma_h = 48.0 \text{ MPa} \quad \longleftarrow$$

(c) Tensile stress in welds (Eq. 8-6)

$$\sigma_w = \frac{pr}{2t} = 24.0 \text{ MPa} \quad \longleftarrow$$

(d) Maximum shear stress in hemispheres (Eq. 8-3)

$$\tau_h = \frac{pr}{4t} = \frac{\sigma_h}{2} = 12.0 \text{ MPa} \quad \longleftarrow$$

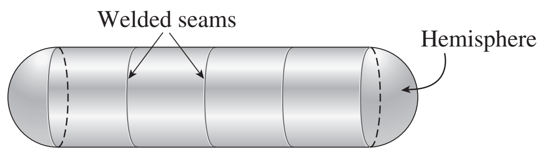
(e) Maximum shear stress in cylinder (Eq. 8-10)

$$\tau_c = \frac{pr}{2t} = \frac{\sigma_c}{2} = 24.0 \text{ MPa} \quad \longleftarrow$$

**Problem 8.3-11** A cylindrical tank with diameter $d = 16$ in. is subjected to internal gas pressure $p = 400$ psi. The tank is constructed of steel sections that are welded circumferentially (see figure). The heads of the tank are hemispherical. The allowable tensile and shear stresses are 8000 psi and 3200 psi, respectively. Also, the allowable tensile stress perpendicular to a weld is 6400 psi.

Determine the minimum required thickness $t_{min}$ of (a) the cylindrical part of the tank, and (b) the hemispherical heads.

**Solution 8.3-11 Cylindrical tank**




$d = 16.0$ in. $r = 8.0$ in. $p = 400$ psi
$\sigma_{allow} = 8000$ psi (tension)
$\tau_{allow} = 3200$ psi (shear)
Weld: $\sigma_{allow} = 6400$ psi (tension)

(a) FIND MINIMUM THICKNESS OF CYLINDER

TENSION: $\sigma_{max} = \frac{pr}{t}$ (EQ. 8-5)

$$t_{min} = \frac{pr}{\sigma_{allow}} = \frac{(400 \text{ psi})(8.0 \text{ in.})}{8000 \text{ psi}} = 0.40 \text{ in.}$$

SHEAR: $\tau_{max} = \frac{pr}{2t}$ (EQ. 8-10)

$$t_{min} = \frac{pr}{2\tau_{allow}} = \frac{(400 \text{ psi})(8.0 \text{ in.})}{(2)(3200 \text{ psi})} = 0.50 \text{ in.}$$

WELD: $\sigma = \frac{pr}{2t}$ (EQ. 8-6)

$$t_{min} = \frac{pr}{2\sigma_{allow}} = \frac{(400 \text{ psi})(8.0 \text{ in.})}{2(6400 \text{ psi})} = 0.25 \text{ in.}$$

Shear governs. $t_{min} = 0.50$ in. ←

(b) FIND MINIMUM THICKNESS OF HEMISPHERES

TENSION: $\sigma_{max} = \frac{pr}{2t}$ (EQ. 8-1)

$$t_{min} = \frac{pr}{2\sigma_{allow}} = \frac{(400 \text{ psi})(8.0 \text{ in.})}{2(8000 \text{ psi})} = 0.20 \text{ in.}$$

SHEAR: $\tau_{max} = \frac{pr}{4t}$ (EQ. 8-3)

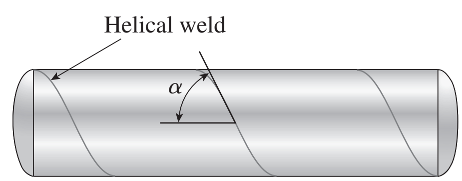
$$t_{min} = \frac{pr}{4\tau_{allow}} = \frac{(400 \text{ psi})(8.0 \text{ in.})}{4(3200 \text{ psi})} = 0.25 \text{ in.}$$

Shear governs. $t_{min} = 0.25$ in. ←

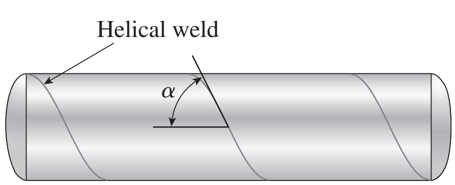
---

**Problem 8.3-12** A pressurized steel tank is constructed with a helical weld that makes an angle $\alpha = 60°$ with the longitudinal axis (see figure). The tank has radius $r = 0.5$ m, wall thickness $t = 15$ mm, and internal pressure $p = 2.4$ MPa. Also, the steel has modulus of elasticity $E = 200$ GPa and Poisson's ratio $\nu = 0.30$.

Determine the following quantities for the cylindrical part of the tank: (a) the circumferential and longitudinal stresses, (b) the maximum in-plane and out-of-plane shear stresses, (c) the circumferential and longitudinal strains, and (d) the normal and shear stresses acting on planes parallel and perpendicular to the weld (show these stresses on a properly oriented stress element).




**Solution 8.3-12 Cylindrical pressure vessel**




$\alpha = 60°$ $r = 0.5$ m
$t = 15$ mm $p = 2.4$ MPa $E = 200$ GPa
$\nu = 0.30$

(a) CIRCUMFERENTIAL STRESS

$$\sigma_1 = \frac{pr}{t} = \frac{(24 \text{ MPa})(0.5 \text{ m})}{(15 \text{ mm})} = 80 \text{ MPa} \leftarrow$$

LONGITUDINAL STRESS

$$\sigma_2 = \frac{pr}{2t} = \frac{\sigma_1}{2} = 40 \text{ MPa} \leftarrow$$

(b) IN-PLANE SHEAR STRESS

$$\tau_1 = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t} = 20 \text{ MPa} \leftarrow$$

OUT-OF-PLANE SHEAR STRESS

$$\tau_2 = \frac{\sigma_1}{2} = \frac{pr}{2t} = 40 \text{ MPa} \leftarrow$$

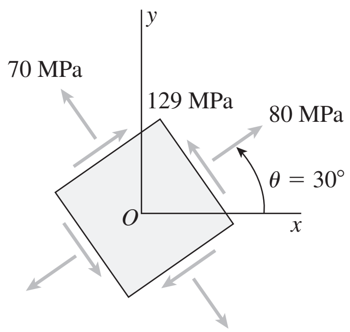
(c) CIRCUMFERENTIAL STRAIN $\varepsilon_1 = \frac{\sigma_1}{2E}(2 - \nu)$

$$\varepsilon_1 = \frac{80 \text{ MPa}}{2(200 \text{ GPa})}(2 - 0.30) = 340 \times 10^{-6} \quad \longleftarrow$$

LONGITUDINAL STRAIN $\varepsilon_2 = \frac{\sigma_2}{E}(1 - 2\nu)$

$$\varepsilon_2 = \frac{(40 \text{ MPa})}{(200 \text{ GPa})}[1 - 2(0.30)] = 80 \times 10^{-6} \quad \longleftarrow$$

(d) STRESSES ON WELD




$\alpha = 60°$ $\quad \theta = 90° - \alpha = 30°$

$\sigma_x = \sigma_2 = 40$ MPa $\quad \sigma_y = \sigma_1 = 80$ MPa

$\tau_{xy} = 0$

FOR $\theta = 30°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 60 \text{ MPa} - (20 \text{ MPa})(\cos 60°) + 0$$

$$= 60 \text{ MPa} - 10 \text{ MPa} = 50 \text{ MPa} \quad \longleftarrow$$

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= +(20 \text{ MPa})(\sin 60°) + 0$$

$$= 17.3 \text{ MPa} \quad \longleftarrow$$

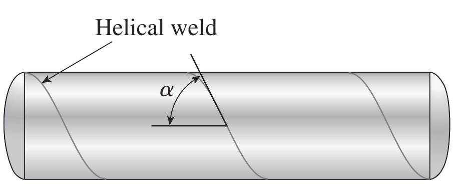
$$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 40 \text{ MPa} + 80 \text{ MPa} - 50 \text{ MPa}$$

$$= 70 \text{ MPa} \quad \longleftarrow$$

---

**Problem 8.3-13** Solve the preceding problem for a welded tank with $\alpha = 65°$, $r = 18$ in., $t = 0.6$ in., $p = 200$ psi, $E = 30 \times 10^6$ psi, and $\nu = 0.30$.

**Solution 8.3-13 Cylindrical tank**




$\alpha = 65°$ $\quad r = 18$ in.

$t = 0.6$ in. $\quad p = 200$ psi $\quad E = 30 \times 10^6$ psi

$\nu = 0.30$

(a) CIRCUMFERENTIAL STRESS

$$\sigma_1 = \frac{pr}{t} = \frac{(200 \text{ psi})(18 \text{ in.})}{(0.6 \text{ in.})} = 6000 \text{ psi} \quad \longleftarrow$$

LONGITUDINAL STRESS

$$\sigma_2 = \frac{pr}{2t} = \frac{\sigma_1}{2} = 3000 \text{ psi} \quad \longleftarrow$$

(b) IN-PLANE SHEAR STRESS

$$\tau_1 = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t} = 1500 \text{ psi} \quad \longleftarrow$$

OUT-OF-PLANE SHEAR STRESS

$$\tau_2 = \frac{\sigma_1}{2} = \frac{pr}{2t} = 3000 \text{ psi} \quad \longleftarrow$$

(c) CIRCUMFERENTIAL STRAIN $\varepsilon_1 = \frac{\sigma_1}{2E}(2 - \nu)$

$$\varepsilon_1 = \frac{6000 \text{ psi}}{2(30 \times 10^6 \text{ psi})}(2 - 0.30)$$

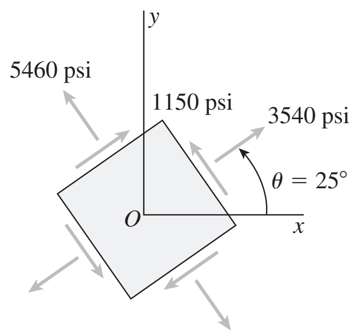
$$= 170 \times 10^{-6} \quad \longleftarrow$$

LONGITUDINAL STRAIN $\varepsilon_2 = \frac{\sigma_2}{E}(1 - 2\nu)$

$$\varepsilon_2 = \frac{3000 \text{ psi}}{30 \times 10^6 \text{ psi}}[1 - 2(0.30)]$$

$$= 40 \times 10^{-6} \quad \longleftarrow$$

(d) STRESS ON WELD




$\alpha = 65°$   $\theta = 90° - \alpha = 25°$

$\sigma_x = \sigma_2 = 3000$ psi   $\sigma_y = \sigma_1 = 6000$ psi

$\tau_{xy} = 0$

FOR $\theta = 25°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= 4500$ psi $-$ (1500 psi)(cos 50°) + 0

$= 4500$ psi $-$ 960 psi $= 3540$ psi ⟵

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= +$(1500 psi)(sin 50°) + 0

$= 1150$ psi ⟵

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 3000$ psi

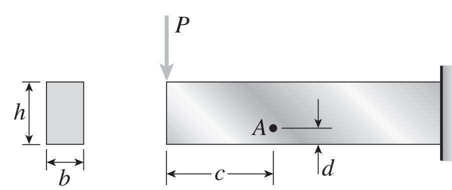
$+ 6000$ psi $-$ 3540 psi $= 5460$ psi ⟵

## Maximum Stresses in Beams

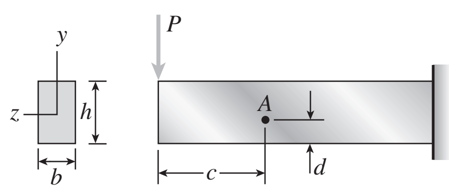
*When solving the problems for Section 8.4, consider only the in-plane stresses and disregard the weights of the beams.*

**Problem 8.4-1** A cantilever beam of rectangular cross section is subjected to a concentrated load $P = 15$ k acting at the free end (see figure). The beam has width $b = 4$ in. and height $h = 10$ in. Point $A$ is located at distance $c = 2$ ft from the free end and distance $d = 3$ in. from the bottom of the beam.

Calculate the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$ at point $A$. Show these stresses on sketches of properly oriented elements.




### Solution 8.4-1 Cantilever beam




$P = 15$ k   $c = 2$ ft $= 24$ in.   $b = 4$ in.

$d = 3$ in.   $h = 10$ in.

STRESSES AT POINT A

$$I = \frac{bh^3}{12} = 333.3 \text{ in.}^4$$

$M = -Pc = -360 \times 10^3$ lb-in.

$V = P = 15{,}000$ lb

$$y_A = -\frac{h}{2} + d = -2.0 \text{ in.}$$

$$\sigma_x = -\frac{My_A}{I} = -\frac{(-360 \times 10^3 \text{ lb-in.})(-2.0 \text{ in.})}{333.3 \text{ in.}^4}$$

$= -2160$ psi

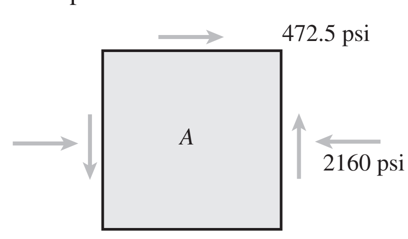
$$\tau = \frac{VQ}{Ib} \qquad Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) = 42.0 \text{ in.}^3$$

$$\tau = \frac{(15{,}000 \text{ lb})(42.0 \text{ in.}^3)}{(333.3 \text{ in.}^4)(4 \text{ in.})} = 472.5 \text{ psi}$$

$\sigma_x = -2160$ psi

$\sigma_y = 0$

$\tau_{xy} = 472.5$ psi



PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -0.4375$$

$2\theta_p = -23.63^\circ$ and $\theta_p = -11.81^\circ$

$2\theta_p = 156.37^\circ$ and $\theta_p = 78.19^\circ$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

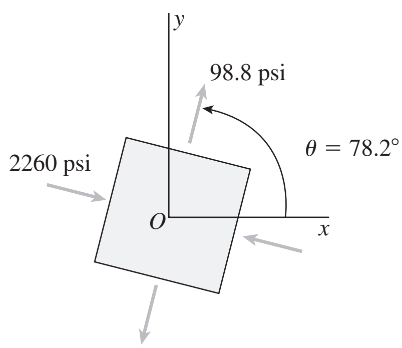
For $2\theta_p = -23.63^\circ$: $\sigma_{x_1} = -2259$ psi

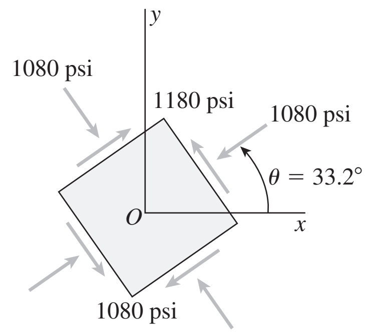
For $2\theta_p = 156.37^\circ$: $\sigma_{x_1} = 98.8$ psi

Therefore,

$\sigma_1 = 99$ psi and $\theta_{p_1} = 78.2^\circ$

$\sigma_2 = -2260$ psi and $\theta_{p_2} = -11.8^\circ$ ←




MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1179 \text{ psi}$$

$\theta_{s_1} = \theta_{p_1} - 45^\circ = 33.2^\circ$

and $\tau = 1180$ psi

$\theta_{s_2} = \theta_{s_1} + 90^\circ = 123.2^\circ$

and $\tau^1 = -1180$ psi ←

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = -1080 \text{ psi}$$

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**Problem 8.4-2** Solve the preceding problem for the following data: $P = 120$ kN, $b = 100$ mm, $h = 200$ mm, $c = 0.5$ m, and $d = 150$ mm.

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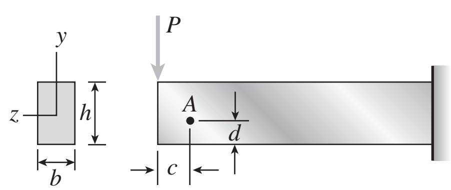
**Solution 8.4-2 Cantilever beam**




$P = 120$ kN  $b = 100$ mm

$h = 200$ mm  $c = 0.5$ m

$d = 150$ mm

STRESSES AT POINT A

$$I = \frac{bh^3}{12} = 66.67 \times 10^6 \text{ mm}^4$$

$M = -Pc = -60{,}000$ N · m

$V = P = 120$ kN

$$y_A = -\frac{h}{2} + d = 50 \text{ mm}$$

$$\sigma_x = -\frac{My_A}{I} = -\frac{(-60{,}000 \text{ N}\cdot\text{m})(50 \text{ mm})}{66.67 \times 10^6 \text{ mm}^4}$$

$= 45.0$ MPa

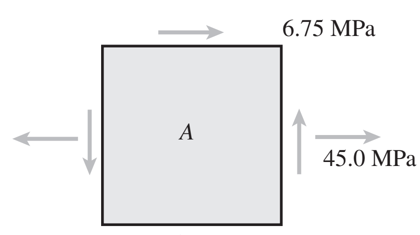
$$\tau = \frac{VQ}{Ib} \qquad Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) = 375{,}000 \text{ mm}^3$$

$$\tau = \frac{(120 \text{ kN})(375{,}000 \text{ mm}^3)}{(66.67 \times 10^6 \text{ mm}^4)(100 \text{ mm})} = 6.75 \text{ MPa}$$

$\sigma_x = 45.0$ MPa

$\sigma_y = 0$

$\tau_{xy} = 6.75$ MPa



PRINCIPAL STRESSES

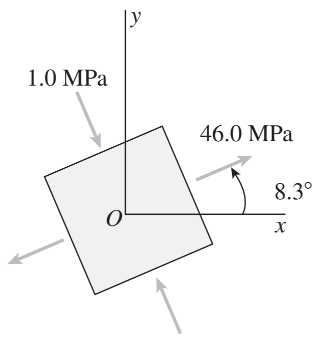
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.3000$$

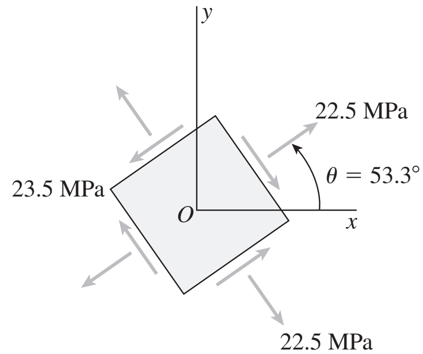
$2\theta_p = 16.70°$ and $\theta_p = 8.35°$
$2\theta_p = 196.7°$ and $\theta_p = 98.35°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

For $2\theta_p = 16.70°$: $\sigma_{x_1} = 45.99$ MPa
For $2\theta_p = 196.70°$: $\sigma_{x_1} = -0.99$ MPa

Therefore,
$\sigma_1 = 46.0$ MPa and $\theta_{p_1} = 83°$
$\sigma_2 = -1.0$ MPa and $\theta_{p_2} = 98.3°$ ←




MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.5 \text{ MPa}$$

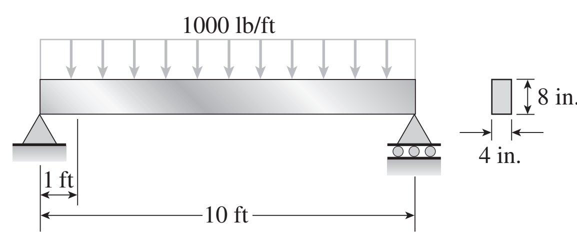
$\theta_{s_1} = \theta_{p_1} - 45° = -36.7°$
and $\tau = 23.5$ MPa
$\theta_{s_2} = \theta_{s_1} + 90° = 53.3°$
and $\tau = -23.5$ MPa ←

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 22.5 \text{ MPa}$$

---

**Problem 8.4-3** A simple beam of rectangular cross section (width 4 in., height 8 in.) carries a uniform load of 1000 lb/ft on a span of 10 ft (see figure).

Find the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$ at a cross section 1 ft from the left-hand support at each of the following locations: (a) the neutral axis, (b) 2 in. above the neutral axis, and (c) the top of the beam. (Disregard the direct compressive stresses produced by the uniform load bearing against the top of the beam.)

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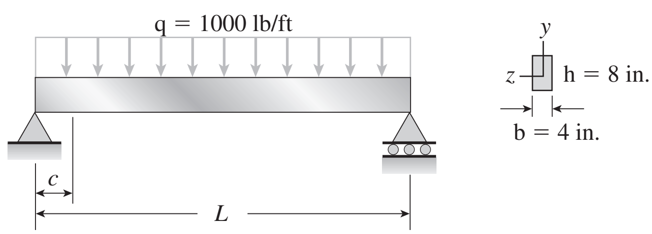
**Solution 8.4-3 Simply supported beam**




$$I = \frac{bh^3}{12} = 170.67 \text{ in.}^4$$

$A = bh = 32$ in.$^2$

$$M = \frac{qLc}{2} - \frac{qc^2}{2} = 54{,}000 \text{ lb-in.}$$

$c = 1.0$ ft $\quad L = 10$ ft $\quad V = \frac{qL}{2} - qc = 4{,}000$ lb

(a) NEUTRAL AXIS

$$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = -\frac{3V}{2A} = -187.5 \text{ psi}$$

Pure shear: $\sigma_1 = 188$ psi, $\sigma_2 = -188$ psi,
$\tau_{max} = 188$ psi ←

(b) 2 in. ABOVE THE NEUTRAL AXIS

$$\sigma_x = -\frac{My}{I} = -\frac{(54{,}000 \text{ lb-in.})(2 \text{ in.})}{170.67 \text{ in.}^4} = -632.8 \text{ psi}$$

$$\sigma_y = 0$$

$$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{(4000 \text{ lb})(4 \text{ in.})(2 \text{ in.})(3 \text{ in.})}{(170.67 \text{ in.}^4)(4 \text{ in.})}$$

$$= -140.6 \text{ psi}$$

From Eq. (8-22):

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = -316.4 \pm 346.2 \text{ psi}$$

$$\sigma_1 = 30 \text{ psi} \qquad \sigma_2 = -663 \text{ psi} \quad \longleftarrow$$

From Eq. (8-24):

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = 346 \text{ psi} \quad \longleftarrow$$

(c) TOP OF THE BEAM

$$\sigma_x = -\frac{Mc}{I} = -\frac{(54{,}000 \text{ lb-in.})(4 \text{ in.})}{170.67 \text{ in.}^4} = -1266 \text{ psi}$$

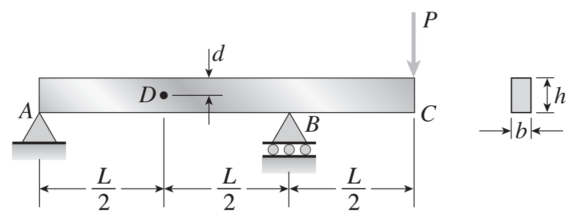
$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

Uniaxial stress: $\sigma_1 = 0$, $\sigma_2 = -1266$ psi,

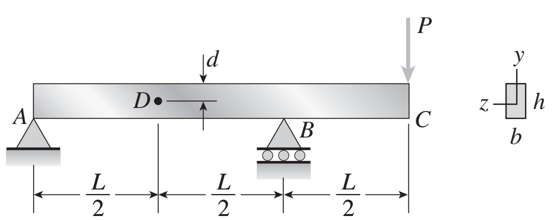
$$\tau_{max} = 633 \text{ psi} \quad \longleftarrow$$

**Problem 8.4-4** An overhanging beam $ABC$ of rectangular cross section supports a concentrated load $P$ at the free end (see figure). The span length from $A$ to $B$ is $L$, and the length of the overhang is $L/2$. The cross section has width $b$ and height $h$. Point $D$ is located midway between the supports at a distance $d$ from the top face of the beam.

Knowing that the maximum tensile stress (principal stress) at point $D$ is $\sigma_1 = 36.1$ MPa, determine the magnitude of the load $P$. Data for the beam are as follows: $L = 1.5$ m, $b = 45$ mm, $h = 180$ mm, and $d = 30$ mm.




**Solution 8.4-4 Overhanging beam**




$L = 1.5$ m $\quad b = 45$ mm $\quad h = 180$ mm
$d = 30$ mm

Maximum principal stress at point $D$:
$\sigma_1 = 36.1$ MPa

Determine the load $P$ units: $P$ = newton

$$M_D = -\frac{PL}{4} = -0.375\,P \text{ (N} \cdot \text{m)}$$

$$V_D = \frac{P}{2} = 0.5\,P \text{ (N)}$$

STRESSES AT POINT $D$

$$I = \frac{bh^3}{12} = 21.87 \times 10^6 \text{ mm}^4 \qquad y = \frac{h}{2} - d = 60 \text{ mm}$$

$$\sigma_x = -\frac{My}{I} = -\frac{(-0.375\,P)(60 \text{ mm})}{21.87 \times 10^6 \text{ mm}^4}$$

$$= 1{,}028.81\,P \text{ (Pa)}$$

$$\sigma_y = 0$$

$$Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) = 101{,}250 \text{ mm}^3$$

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{(0.5\,P)(101{,}250 \text{ mm}^3)}{(21.87 \times 10^6 \text{mm}^4)(45 \text{ mm})}$$

$$= 51.4403\,P \text{ (Pa)}$$

PRINCIPAL STRESS

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= 514.41\,P + \sqrt{(514.41\,P)^2 + (51.4403\,P)^2}$$

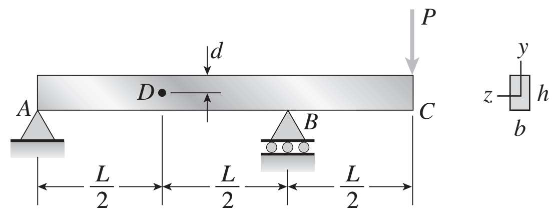
$$= 1031.4\,P \text{ (Pa)}$$

But $\sigma_1 = 36.1$ MPa $\quad \therefore P = 35{,}000$ N

$$P = 35.0 \text{ kN} \quad \longleftarrow$$

**Problem 8.4-5** Solve the preceding problem if the stress and dimensions are as follows: $\sigma_1 = 2320$ psi, $L = 75$ in., $b = 2.0$ in., $h = 9.0$ in., and $d = 1.5$ in.

**Solution 8.4-5 Overhanging beam**




$L = 75$ in. $\quad b = 2.0$ in. $\quad h = 9.0$ in.
$d = 1.5$ in.

Maximum principal stress at point $D$:
$\sigma_1 = 2320$ psi

Determine the load $P$ units: $P$ = pounds

$$M_D = -\frac{PL}{4} = -18.75\,P \text{ (lb-in.)}$$

$$V_D = \frac{P}{2} = 0.5\,P \text{ (lb)}$$

STRESSES AT POINT $D$

$$I = \frac{bh^3}{12} = 121.5 \text{ in.}^4 \qquad y = \frac{h}{2} - d = 3.0 \text{ in.}$$

$$\sigma_x = -\frac{My}{I} = -\frac{(-18.75\,P)(3.0 \text{ in.})}{121.5 \text{ in.}^4}$$
$$= 0.462963\,P \text{ (psi)}$$

$$\sigma_y = 0$$

$$Q = bd\left(\frac{h}{2} - \frac{d}{2}\right) = 11.25 \text{ in.}^3$$

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{(0.5\,P)(11.25 \text{ in.}^3)}{(121.5 \text{ in.}^4)(2.0 \text{ in.})}$$
$$= 0.0231481\,P \text{ (psi)}$$

PRINCIPAL STRESS

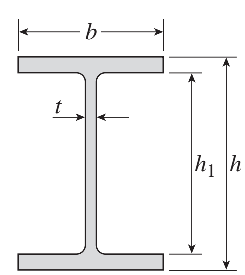
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = 0.231482\,P$$
$$+ \sqrt{(0.231482\,P)^2 + (0.0231481\,P)^2}$$
$$= 0.46412\,P \text{ (psi)}$$

But $\sigma_1 = 2320$ psi $\quad \therefore P = 4999$ lb

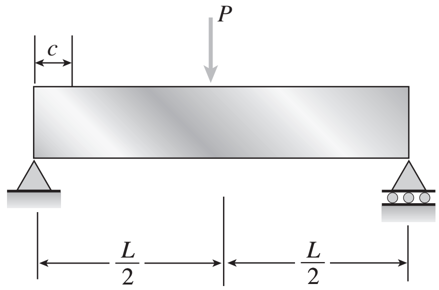
$P = 5.00$ k ←

**Problem 8.4-6** A beam of wide-flange cross section (see figure) has the following dimensions: $b = 120$ mm, $t = 10$ mm, $h = 300$ mm, and $h_1 = 260$ mm. The beam is simply supported with span length $L = 3.0$ m. A concentrated load $P = 120$ kN acts at the midpoint of the span.

At a cross section located 1.0 m from the left-hand support, determine the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$ at each of the following locations: (a) the top of the beam, (b) the top of the web, and (c) the neutral axis.



## Solution 8.4-6 Simply supported beam




$P = 120\text{ kN} \quad L = 3.0\text{ m}$

$c = 1.0\text{ m} \quad M = \frac{Pc}{2} = 60\text{ kN}\cdot\text{m}$

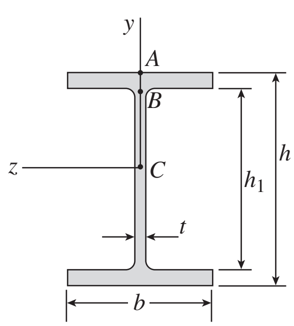
$V = \frac{P}{2} = 60\text{ kN}$

$b = 120\text{ mm} \quad t = 10\text{ mm} \quad h = 300\text{ mm}$

$h_1 = 260\text{ mm}$

$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = 108.89 \times 10^6\text{ mm}^4$$

(a) Top of the beam (point A)




$$\sigma_x = -\frac{Mc}{I} = -\frac{(60\text{ kN}\cdot\text{m})(150\text{ mm})}{108.89 \times 10^6\text{ mm}^4}$$

$= -82.652\text{ MPa}$

$\sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 0$

$\sigma_2 = -82.7\text{ MPa} \quad \tau_{max} = 41.3\text{ MPa}$ ←

(b) Top of the web (point B)

$$\sigma_x = -\frac{My}{I} = -\frac{(60\text{ kN}\cdot\text{m})(130\text{ mm})}{108.89 \times 10^6\text{ mm}^4}$$

$= -71.63\text{ MPa}$

$\sigma_y = 0$

$$Q = (b)\left(\frac{h - h_1}{2}\right)\left(\frac{h + h_1}{4}\right) = 336 \times 10^3\text{ mm}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(60\text{ kN})(336 \times 10^3\text{ mm}^3)}{(108.89 \times 10^6\text{ mm}^4)(10\text{ mm})}$$

$= -18.51\text{ MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$= -35.82 \pm 40.32\text{ MPa}$

$\sigma_1 = 4.5\text{ MPa}, \sigma_2 = -76.1\text{ MPa}$ ←

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 40.3\text{ MPa}$$ ←

(c) Neutral axis (point C) $\sigma_x = 0 \quad \sigma_y = 0$

$$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b - t)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right)$$

$= 420.5 \times 10^3\text{ mm}^3$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(60\text{ kN})(420.5 \times 10^3\text{ mm}^3)}{(108.89 \times 10^6\text{ mm}^4)(10\text{ mm})}$$

$= -23.17\text{ MPa}$

Pure shear: $\sigma_1 = 23.2\text{ MPa},$

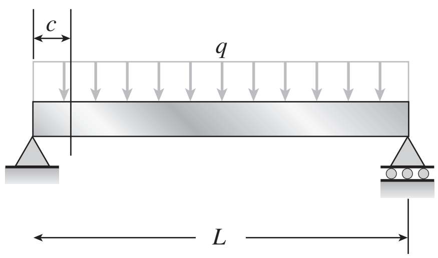
$\sigma_2 = -23.2\text{ MPa}$

$\tau_{max} = 23.2\text{ MPa}$ ←

**Problem 8.4-7** A beam of wide-flange cross section (see figure) has the following dimensions: $b = 5$ in., $t = 0.5$ in., $h = 12$ in., and $h_1 = 10.5$ in. The beam is simply supported with span length $L = 10$ ft and supports a uniform load $q = 6$ k/ft.

Calculate the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$ at a cross section located 3 ft from the left-hand support at each of the following locations: (a) the bottom of the beam, (b) the bottom of the web, and (c) the neutral axis.

**Solution 8.4-7 Simply supported beam**




$q = 6.0$ k/ft $\quad L = 10$ ft $= 120$ in.

$$c = 3 \text{ ft} = 36 \text{ in.} \qquad M = \frac{qLc}{2} - \frac{qc^2}{2} = 756{,}000 \text{ lb-in.}$$

$$V = \frac{qL}{2} - qc = 12{,}000 \text{ lb}$$

$b = 5.0$ in. $\quad t = 0.5$ in. $\quad h = 12$ in.
$h_1 = 10.5$ in.

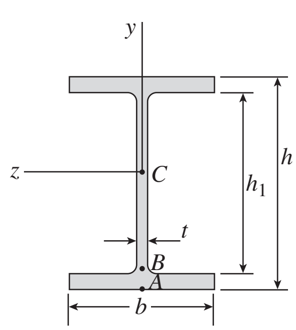
$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = 285.89 \text{ in.}^4$$

(a) Bottom of the beam (point A)

$$\sigma_x = -\frac{Mc}{I} = -\frac{(756{,}000 \text{ lb-in.})(-6.0 \text{ in.})}{285.89 \text{ in.}^4}$$
$$= 15{,}866 \text{ psi}$$

$\sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 15{,}870$ psi, $\sigma_2 = 0 \quad \tau_{max} = 7930$ psi ←




(b) Bottom of the web (point B)

$$\sigma_x = -\frac{My}{I} = -\frac{(756{,}000 \text{ lb-in.})(-5.25 \text{ in.})}{285.89 \text{ in.}^4}$$
$$= 13{,}883 \text{ psi}$$

$$\sigma_y = 0 \qquad Q = b\left(\frac{h - h_1}{2}\right)\left(\frac{h + h_1}{4}\right) = 21.094 \text{ in.}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(12{,}000 \text{ lb})(21.094 \text{ in.}^3)}{(285.89 \text{ in.}^4)(0.5 \text{ in.})}$$
$$= -1771 \text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= 6941.5 \pm 7163.9 \text{ psi}$$

$\sigma_1 = 14{,}100$ psi, $\sigma_2 = -220$ psi ←

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7160 \text{ psi}$$ ←

(c) Neutral axis (point C)

$\sigma_x = 0 \quad \sigma_y = 0$

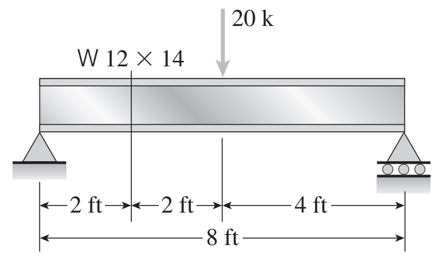
$$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b - t)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right)$$
$$= 27.984 \text{ in.}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(12{,}000 \text{ lb})(27.984 \text{ in.}^3)}{(285.89 \text{ in.}^4)(0.5 \text{ in.})}$$
$$= -2349 \text{ psi}$$

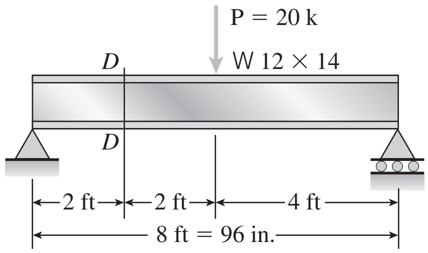
Pure shear: $\sigma_1 = 2350$ psi, $\sigma_2 = -2350$ psi, $\tau_{max} = 2350$ psi ←

**Problem 8.4-8** A W 12 × 14 wide-flange beam (see Table E-l, Appendix E) is simply supported with a span length of 8 ft (see figure). The beam supports a concentrated load of 20 kips at midspan.

At a cross section located 2 ft from the left-hand support, determine the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$ at each of the following locations: (a) the top of the beam, (b) the top of the web, and (c) the neutral axis.



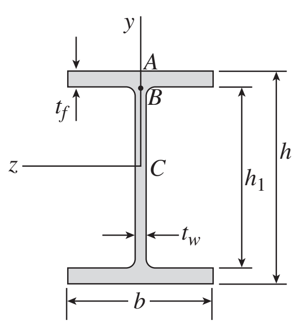

## Solution 8.4-8 Simply supported beam




At Section D-D:

$$M = \frac{P}{2}(2\text{ ft}) = 20\text{ k-ft} = 240\text{ k-in.}$$

$$V = \frac{P}{2} = 10\text{ k}$$




W12 × 14
$I = 88.6\text{ in.}^4$
$b = 3.970\text{ in.}$ $\quad t_w = 0.200\text{ in.}$
$t_f = 0.225\text{ in.}$ $\quad h = 11.91\text{ in.}$
$h_1 = h - 2t_f = 11.460\text{ in.}$

(a) TOP OF THE BEAM (POINT A)

$$\sigma_x = -\frac{Mc}{I} = -\frac{M(h/2)}{I} = -\frac{(240\text{ k-in.})(5.955\text{ in.})}{88.6\text{ in.}^4}$$

$$= -16{,}130\text{ psi}$$

$\sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 0, \quad \sigma_2 = -16{,}130\text{ psi},$

$$\tau_{max} = \left|\frac{\sigma_x}{2}\right| = 8{,}070\text{ psi} \quad \longleftarrow$$

(b) TOP OF THE WEB (POINT B)

$$\sigma_x = -\frac{My}{I} = -\frac{M(h_1/2)}{I} = -15{,}520\text{ psi}$$

$$\sigma_y = 0 \qquad Q = (b)\left(\frac{h - h_1}{2}\right)\left(\frac{h + h_1}{4}\right) = 5.219\text{ in.}^3$$

$$\tau_{xy} = -\frac{VQ}{It_w} = -\frac{(10\text{ k})(5.219\text{ in.}^3)}{(88.6\text{ in.}^4)(0.200\text{ in.})}$$

$$= -2950\text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -7{,}760\text{ psi} \pm 8{,}300\text{ psi}$$

$$\sigma_1 = 540\text{ psi}, \sigma_2 = -16{,}060\text{ psi} \quad \longleftarrow$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 8{,}300\text{ psi} \quad \longleftarrow$$

(c) NEUTRAL AXIS (POINT C)

$\sigma_x = 0 \quad \sigma_y = 0$

$$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b - t_w)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right)$$

$$= 8.502\text{ in.}^3$$

$$\tau_{xy} = -\frac{VQ}{It_w} = -\frac{(10\text{ k})(8.502\text{ in.}^3)}{(88.6\text{ in.}^4)(0.200\text{ in.})}$$

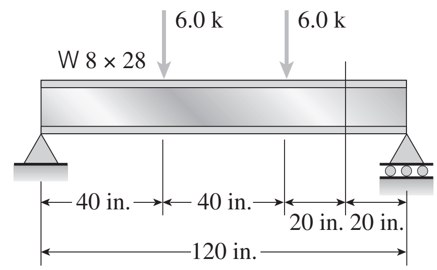
$$= -4{,}800\text{ psi}$$

Pure shear: $\sigma_1 = |\tau_{xy}| \quad \sigma_2 = -\sigma_1 \quad \tau_{max} = |\tau_{xy}|$

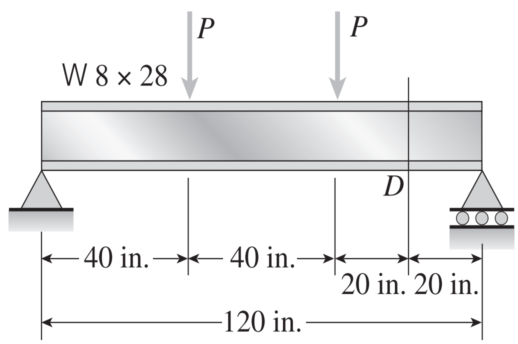
$$\sigma_1 = 4{,}800\text{ psi}, \sigma_2 = -4{,}800\text{ psi}, \tau_{max} = 4{,}800\text{ psi} \quad \longleftarrow$$

**Problem 8.4-9** A W 8 × 28 wide-flange beam (see Table E-l, Appendix E) is simply supported with a span length of 120 in. (see figure). The beam supports two symmetrically placed concentrated loads of 6.0 k each.

At a cross section located 20 in. from the right-hand support, determine the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$ at each of the following locations: (a) the top of the beam, (b) the top of the web, and (c) the neutral axis.




**Solution 8.4-9** **Simply supported beam**

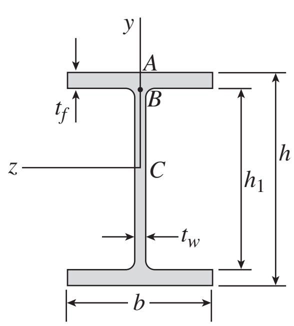



$P = 6.0$ k

At Section D-D:

$M = P(20 \text{ in.}) = 120$ k-in.

$V = -P = -6.0$ k




W 8 × 28

$I = 98.0 \text{ in.}^4$

$b = 6.535$ in. $\quad t_w = 0.285$ in.

$t_f = 0.465$ in. $\quad h = 8.06$ in.

$h_1 = h - 2t_f = 7.13$ in.

(a) Top of the beam (point A)

$$\sigma_x = -\frac{Mc}{I} = -\frac{M(h/2)}{I} = -\frac{(120 \text{ k-in.})(4.03 \text{ in.})}{98.0 \text{ in.}^4}$$

$$= -4435 \text{ psi}$$

$\sigma_y = 0 \qquad \tau_{xy} = 0$

Uniaxial stress:

$\sigma_1 = 0, \quad \sigma_2 = -4930$ psi,

$$\tau_{max} = \left|\frac{\sigma_x}{2}\right| = 2470 \text{ psi} \quad \longleftarrow$$

(b) Top of the web (point B)

$$\sigma_x = -\frac{My}{I} = -\frac{M(h_1/2)}{I} = -\frac{(120 \text{ k-in.})(3.565 \text{ in.})}{98.0 \text{ in.}^4}$$

$$= -4365 \text{ psi}$$

$$\sigma_y = 0 \qquad Q = (b)\left(\frac{h - h_1}{2}\right)\left(\frac{h + h_1}{4}\right) = 11.540 \text{ in.}^3$$

$$\tau_{xy} = -\frac{VQ}{It_w} = -\frac{(-6.0 \text{ k})(11.540 \text{ in.}^3)}{(98.0 \text{ in.}^4)(0.285 \text{ in.})}$$

$$= 2479 \text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -2182 \text{ psi} \pm 3303 \text{ psi}$$

$$\sigma_1 = 1120 \text{ psi}, \sigma_2 = -5480 \text{ psi} \quad \longleftarrow$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 3300 \text{ psi} \quad \longleftarrow$$

(c) Neutral axis (point C)

$\sigma_x = 0 \qquad \sigma_y = 0$

$$Q = b\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) - (b - t_w)\left(\frac{h_1}{2}\right)\left(\frac{h_1}{4}\right)$$

$$= 13.351 \text{ in.}^3$$

$$\tau_{xy} = -\frac{VQ}{It_w} = -\frac{(-6.0 \text{ k})(13.351 \text{ in.}^3)}{(98.0 \text{ in.}^4)(0.285 \text{ in.})}$$

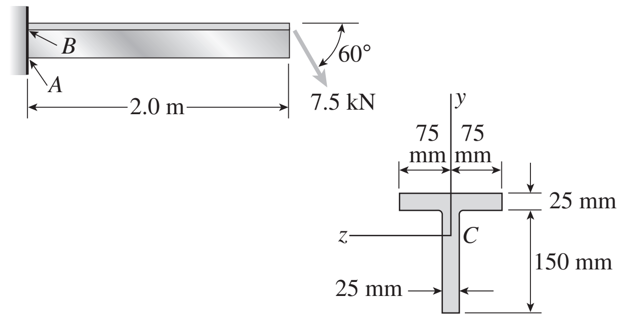
$$= 2870 \text{ psi}$$

Pure shear: $\sigma_1 = |\tau_{xy}| \quad \sigma_2 = -\sigma_1 \quad \tau_{max} = |\tau_{xy}|$

$$\sigma_1 = 2870 \text{ psi}, \ \sigma_2 = -2870 \text{ psi}, \ \tau_{max} = 2870 \text{ psi} \quad \longleftarrow$$

**Problem 8.4-10** A cantilever beam of T-section is loaded by an inclined force of magnitude 7.5 kN (see figure). The line of action of the force is inclined at an angle of 60° to the horizontal and intersects the top of the beam at the end cross section. The beam is 2.0 m long and the cross section has the dimensions shown.

Determine the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$ at points *A* and *B* in the web of the beam.

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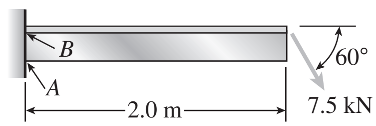
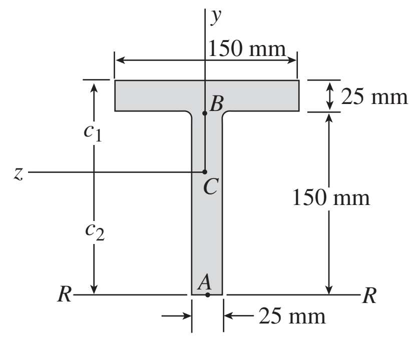
**Solution 8.4-10** **Cantilever beam of T-section**




$P = 7.5$ kN
$L = 2.0$ m
$A = 2(150 \text{ mm})(25 \text{ mm}) = 7500 \text{ mm}^2$
$b = 150$ mm
$t = 25$ mm




LOCATION OF CENTROID C

From Eq. (12-7b) in Chapter 12:

$$c_2 = \frac{\sum \bar{y}_i A_i}{A}$$

Use the base of the cross section as the reference axis (line R-R)

For the web: $\bar{y}A = (75 \text{ mm})(25 \text{ mm})(150 \text{ mm}) = 281{,}250 \text{ mm}^3$

For the flange: $\bar{y}A = (162.5 \text{ mm})(150 \text{ mm})(25 \text{ mm}) = 609{,}375 \text{ mm}^3$

$$c_2 = \frac{281{,}250 \text{ mm}^3 + 609{,}375 \text{ mm}^3}{7500 \text{ mm}^2} = 118.75 \text{ mm}$$

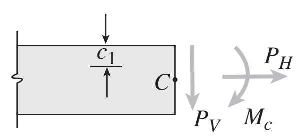
$$c_1 = 175 \text{ mm} - c_2 = 56.25 \text{ mm}$$

MOMENT OF INERTIA

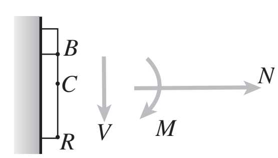
$$I_z = \frac{1}{3}tc_2^3 + \frac{1}{3}bc_1^3 - \frac{1}{3}(b-t)(c_1-t)^3$$
$$= 21.582 \times 10^6 \text{ mm}^4$$

EQUIVALENT LOADS AT FREE END OF BEAM

$P_H = P \cos 60° = 3.75$ kN
$P_V = P \sin 60° = 6.4952$ kN
$M_C = P_H c_1 = 210.94$ N · m




STRESS RESULTANTS AT FIXED END OF BEAM




$N = P_H = 3.75$ kN
$V = P_V = 6.4952$ kN
$M = -M_c - P_V L = -13{,}201$ N · m
(Note that $M$ is negative)

STRESS AT POINT A (BOTTOM OF WEB)

$$\sigma_x = \frac{N}{A} + \frac{Mc_z}{I_z}$$

$$= 0.50 \text{ MPa} - 72.64 \text{ MPa} = -72.14 \text{ MPa}$$

$\sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress:

$\sigma_1 = 0, \quad \sigma_2 = -72.1 \text{ MPa},$ ←

$\tau_{max} = \left|\frac{\sigma_x}{2}\right| = 36.1 \text{ MPa}$

STRESS AT POINT B (TOP OF WEB)

$$\sigma_x = \frac{N}{A} - \frac{M(c_1 - t)}{I_z}$$

$$= 0.50 \text{ MPa} + 19.11 \text{ MPa} = 19.61 \text{ MPa}$$

$$\sigma_y = 0 \qquad Q = bt\left(c_1 - \frac{t}{2}\right) = 164.06 \times 10^3 \text{ mm}^3$$

$$\tau_{xy} = -\frac{VQ}{I_z t} = -\frac{(6.4952 \text{ kN})(164.06 \times 10^3 \text{ mm}^3)}{(21.582 \times 10^6 \text{ mm}^4)(25 \text{ mm})}$$

$$= -1.97 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

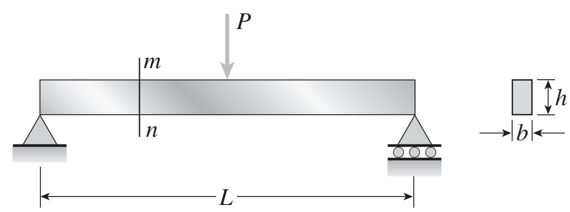
$$= 9.81 \text{ MPa} \pm 10.00 \text{ MPa}$$

$\sigma_1 = 19.8 \text{ MPa}, \sigma_2 = -0.2 \text{ MPa}$ ←

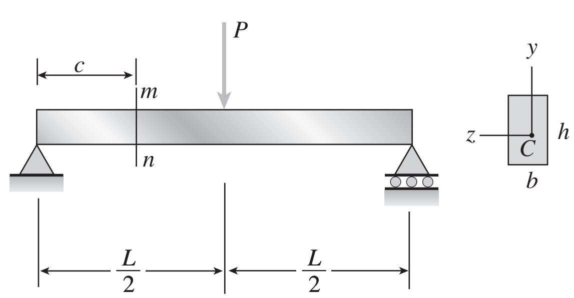
$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 10.0 \text{ MPa}$ ←

**Problem 8.4-11** A simple beam of rectangular cross section has span length $L = 60$ in. and supports a concentrated load $P = 18$ k at the midpoint (see figure). The height of the beam is $h = 6$ in. and the width is $b = 2$ in.

Plot graphs of the principal stresses $\sigma_1$ and $\sigma_2$ and the maximum shear stress $\tau_{max}$, showing how they vary over the height of the beam at cross section $mn$, which is located 20 in. from the left-hand support.




**Solution 8.4-11** **Simple beam**




$P = 18$ k $\quad L = 60$ in. $\quad c = 20$ in. $\quad b = 2$ in. $h = 6$ in.

CROSS SECTION mn

$$M = \frac{Pc}{2} = 180 \text{ k-in.} \qquad V = \frac{P}{2} = 9 \text{ k}$$

$$\sigma_x = -\frac{My}{I} = -\frac{12\,My}{bh^3} = -5000\,y \qquad (1)$$

Units: $y$ = in., $\sigma_x$ = psi

$$\sigma_y = 0 \qquad (2)$$

$$Q = b\left(\frac{h}{2} - y\right)\left(\frac{1}{2}\right)\left(\frac{h}{2} + y\right) = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

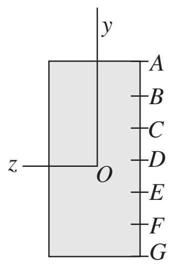
$$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right) = -125(9 - y^2) \qquad (3)$$

Units: $y$ = in., $\tau_{xy}$ = psi

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \qquad (4)$$

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \qquad (5)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \qquad (6)$$




$$y_A = -y_G = \frac{h}{2} = 3 \text{ in.}$$

$$y_B = -y_F = \frac{h}{3} = 2 \text{ in.}$$

$$y_C = -y_E = \frac{h}{6} = 1 \text{ in.}$$

$$y_D = 0$$

POINT A ($y$ = 3 in.)

Eq. (1): $\sigma_x = -15{,}000$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress:
$\sigma_1 = 0$, $\sigma_2 = -15{,}000$ psi, $\tau_{max} = 7500$ psi

POINT B ($y$ = 12 in.)

Eq. (1): $\sigma_x = -10{,}000$ psi $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -625$ psi
Eqs. (4), (5), and (6): $\sigma_1 = 40$ psi,
$\sigma_2 = -10{,}040$ psi, $\tau_{max} = 5040$ psi

POINT C ($y$ = 1 in.)

Eq. (1): $\sigma_x = -5000$ psi $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -1000$ psi
Eqs. (4), (5), and (6): $\sigma_1 = 190$ psi, $\sigma_2 = -5190$ psi,
$\tau_{max} = 2690$ psi

POINT D ($y$ = 0)

$$\sigma_x = 0, \quad \sigma_y = 0,\ \tau_{xy} = -\frac{3V}{2A} = \frac{-3V}{2bh} = -1125 \text{ psi}$$

Pure shear: $\sigma_1 = 1125$ psi, $\sigma_2 = -1125$ psi,
$\tau_{max} = 1125$ psi

POINT E ($y$ = −1 in.)

Eq. (1): $\sigma_x = 5000$ psi $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -1000$ psi
Eqs. (4), (5), and (6): $\sigma_1 = 5190$ psi, $\sigma_2 = -190$ psi,
$\tau_{max} = 2690$ psi

POINT F ($y$ = −2 in.)

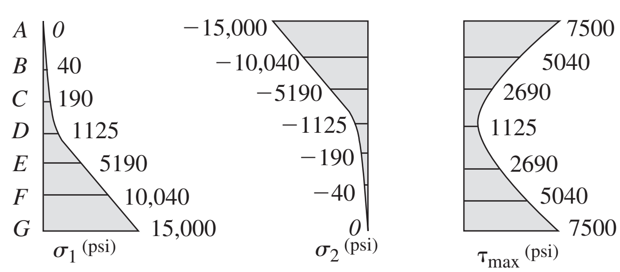
Eq. (1): $\sigma_x = 10{,}000$ psi $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -625$ psi
Eqs. (4), (5), and (6): $\sigma_1 = 10{,}040$ psi,
$\sigma_2 = -40$ psi, $\tau_{max} = 5040$ psi

POINT G ($y$ = −3 in.)

Eq. (1): $\sigma_x = 15{,}000$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

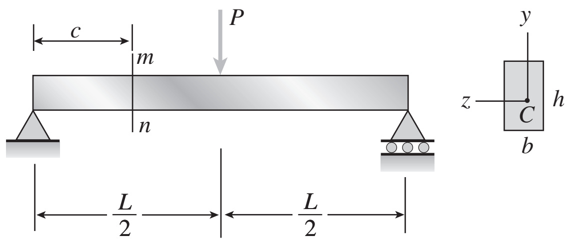
Uniaxial stress: $\sigma_1 = 15{,}000$ psi, $\sigma_2 = 0$,
$\tau_{max} = 7500$ psi

GRAPHS OF STRESSES



**Problem 8.4-12** Solve the preceding problem for a cross section *mn* located 0.15 m from the support if $L = 0.7$ m, $P = 144$ kN, $h = 120$ mm, and $b = 20$ mm.

**Solution 8.4-12 Simple beam**




$P = 144$ kN $\quad L = 0.7$ m $\quad c = 0.15$ m
$b = 20$ mm $\quad h = 120$ mm

CROSS SECTION mn

$$M = \frac{Pc}{2} = 10.8 \text{ kN}\cdot\text{m} \qquad V = \frac{P}{2} = 72 \text{ kN}$$

$$\sigma_x = -\frac{My}{I} = -\frac{12\,My}{bh^3} = -3.75\,y \tag{1}$$

Units: $y$ = mm, $\sigma_x$ = MPa

$$\sigma_y = 0 \tag{2}$$

$$Q = b\left(\frac{h}{2} - y\right)\left(\frac{1}{2}\right)\left(\frac{h}{2} + y\right) = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

$$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

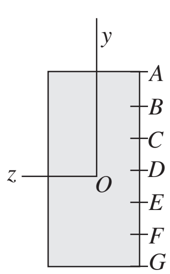
$$= -12.5\left(3.6 - \frac{y^2}{10^3}\right) \tag{3}$$

Units: $y$ = mm, $\tau_{xy}$ = MPa

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \tag{4}$$

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \tag{5}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \tag{6}$$




$$y_A = -y_G = \frac{h}{2} = 60 \text{ mm}$$

$$y_B = -y_F = \frac{h}{3} = 40 \text{ mm}$$

$$y_C = -y_E = \frac{h}{6} = 20 \text{ mm}$$

$$y_D = 0$$

POINT A ($y = 60$ mm)

Eq. (1): $\sigma_x = -225$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress:
$\sigma_1 = 0$, $\sigma_2 = -225$ MPa, $\tau_{\max} = 112$ MPa

POINT B ($y = 40$ mm)

Eq. (1): $\sigma_x = -150$ MPa $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -25$ MPa
Eqs. (4), (5), and (6): $\sigma_1 = 4$ MPa, $\sigma_2 = -154$ MPa, $\tau_{\max} = 79$ MPa

POINT C ($y = 20$ mm)

Eq. (1): $\sigma_x = -75$ MPa $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -40$ MPa
Eqs. (4), (5), and (6): $\sigma_1 = 17$ MPa, $\sigma_2 = -92$ MPa, $\tau_{\max} = 55$ MPa

POINT D ($y = 0$)

$$\sigma_x = 0, \quad \sigma_y = 0, \ \tau_{xy} = -\frac{3V}{2A} = \frac{-3V}{2bh} = -45 \text{ MPa}$$

Pure shear: $\sigma_1 = 45$ MPa, $\sigma_2 = -45$ MPa, $\tau_{\max} = 45$ MPa

POINT E ($y = -20$ mm)

Eq. (1): $\sigma_x = 75$ MPa $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -40$ MPa
Eqs. (4), (5), and (6): $\sigma_1 = 92$ MPa, $\sigma_2 = -17$ MPa, $\tau_{\max} = 55$ MPa

POINT F ($y = -40$ mm)

Eq. (1): $\sigma_x = 150$ MPa $\quad \sigma_y = 0$
Eq. (3): $\tau_{xy} = -25$ MPa
Eqs. (4), (5), and (6): $\sigma_1 = 154$ MPa, $\sigma_2 = -4$ MPa, $\tau_{\max} = 79$ MPa

POINT G ($y = -60$ mm)

Eq. (1): $\sigma_x = 225$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

Uniaxial stress: $\sigma_1 = 225$ MPa, $\sigma_2 = 0$, $\tau_{\max} = 112$ MPa

GRAPHS OF STRESSES